Shear Force and Bending Moment Diagrams

- A ***BEAM*** is a long, slender, structural member designed to support transverse loadings.
- A transverse loading is applied perpendicular to the axis of the beam.
- Beams classified by their supports.

Need to know the internal shear force and the internal bending moment at all locations in the beam. This information is usually presented as a graph or diagram of these values vs. position.

- Bending Moment
- The algebric sum of the moments of the forces on either side of the section of a loaded beam is called Bending Moment.
- SHEAR FORCE
- The algebric sum of the vertical forces on either side of the section of a loaded beam is called Shearing Force

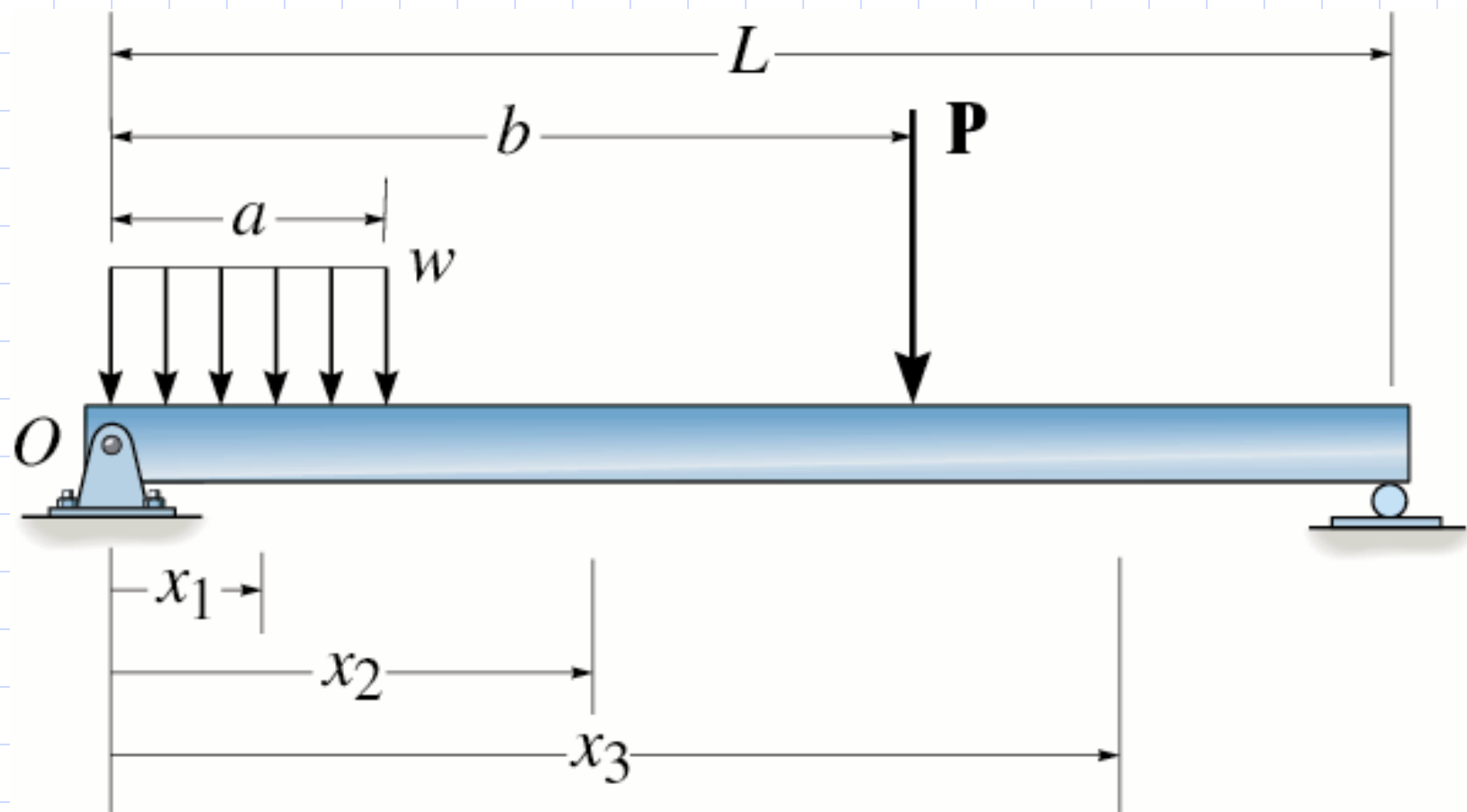
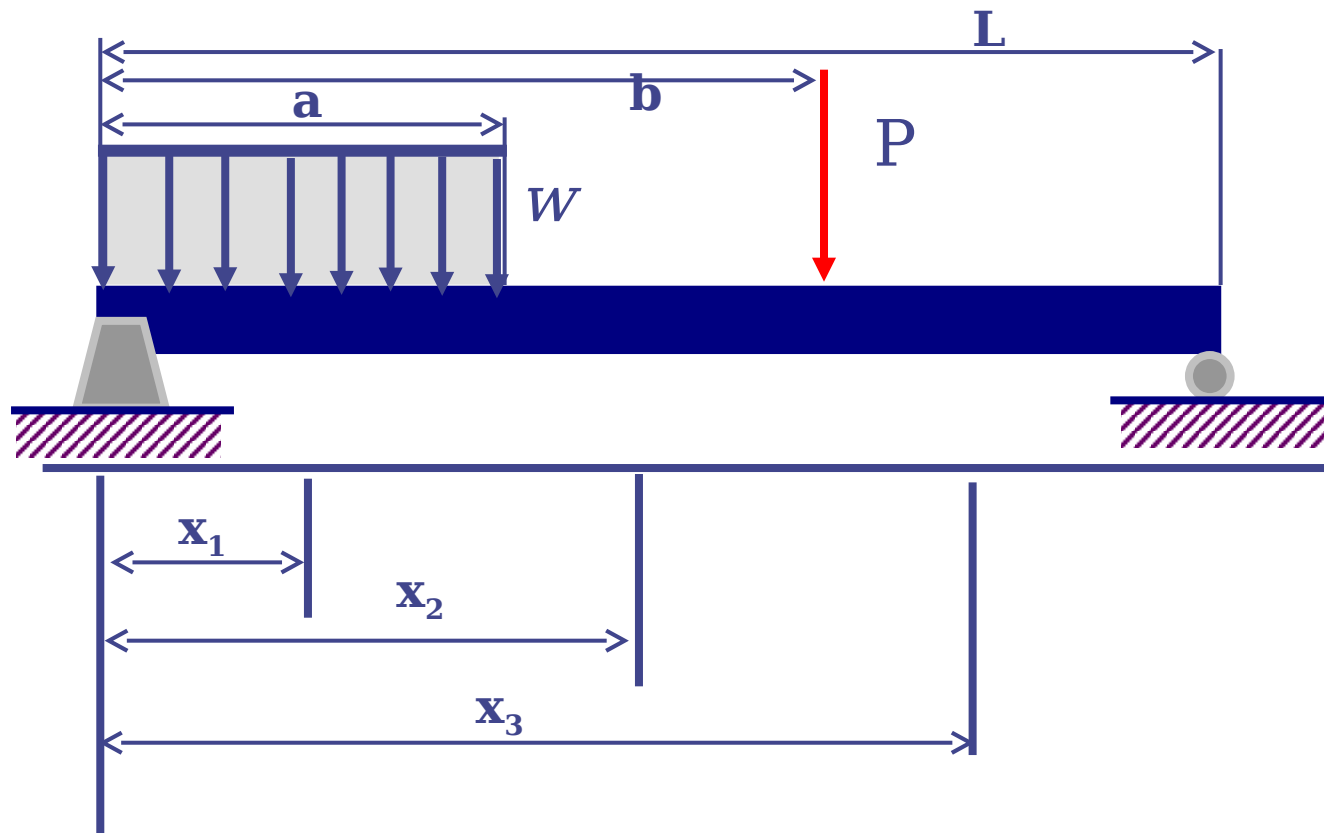
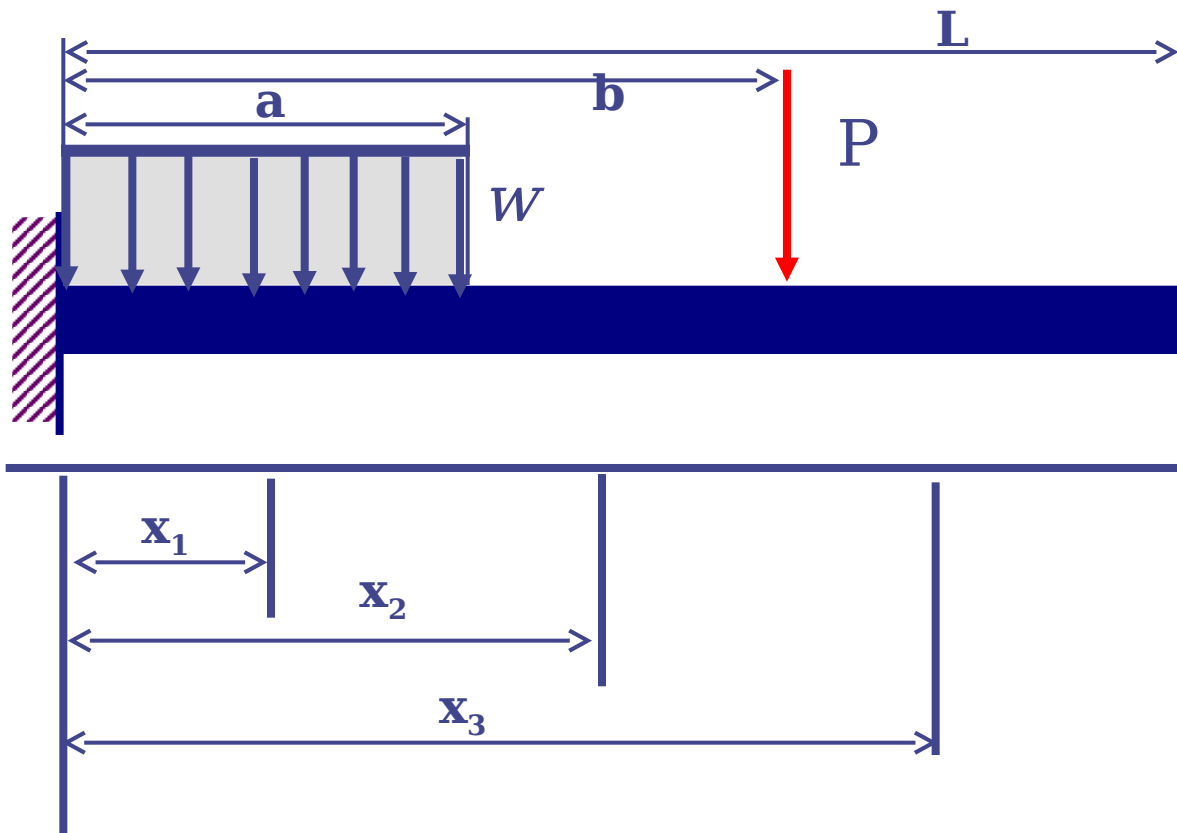



Figure 07.10

Simply Supported Beam

L
b
a
P
W
x1
x2
x3
Cantilever Beam

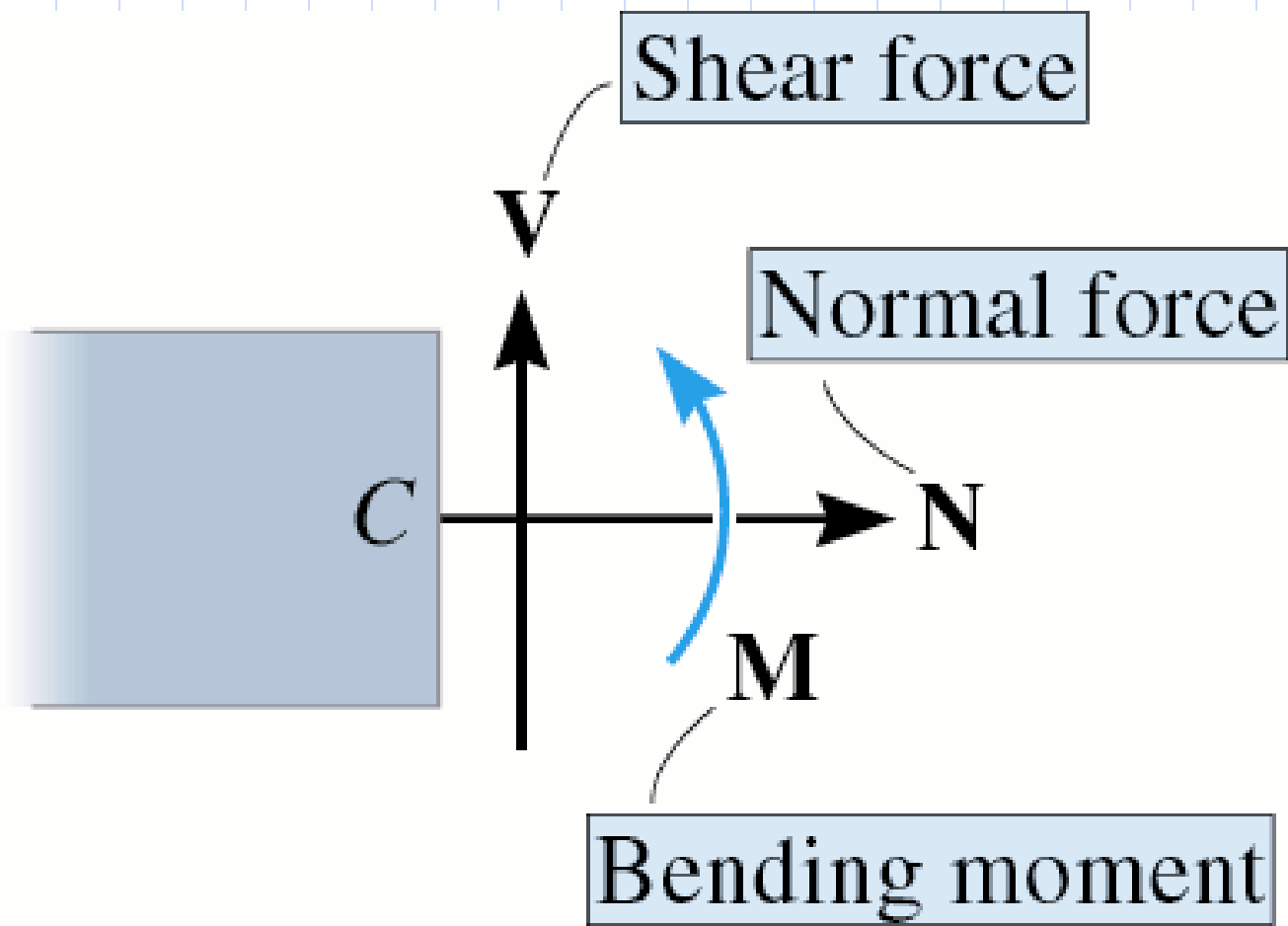
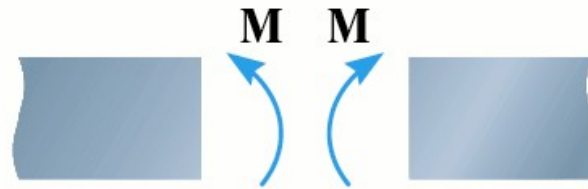
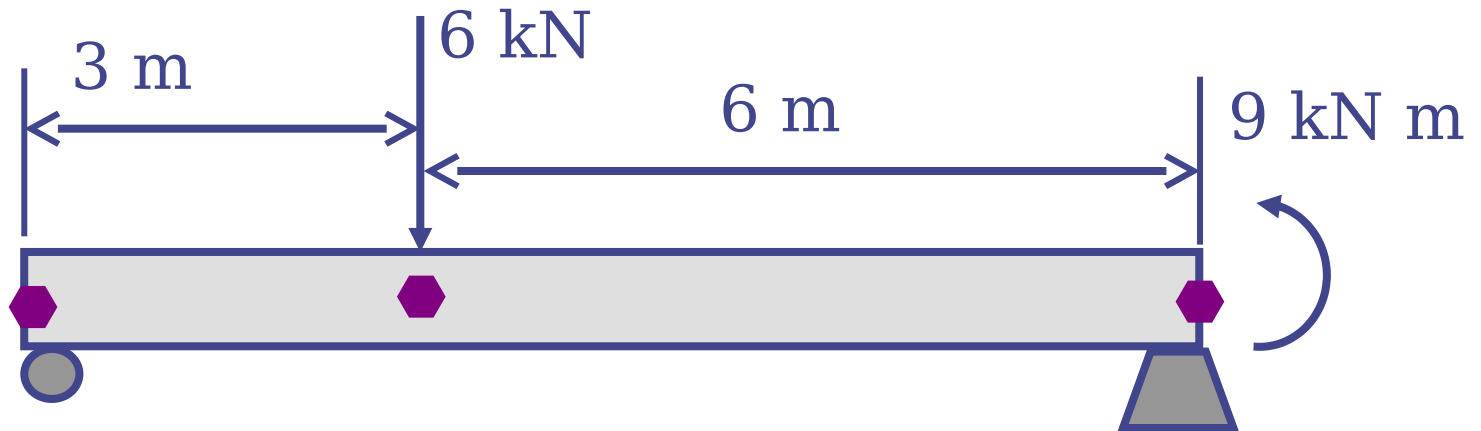
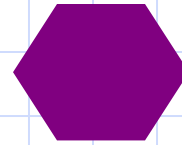



Figure 07.02(a)

Positive Shear

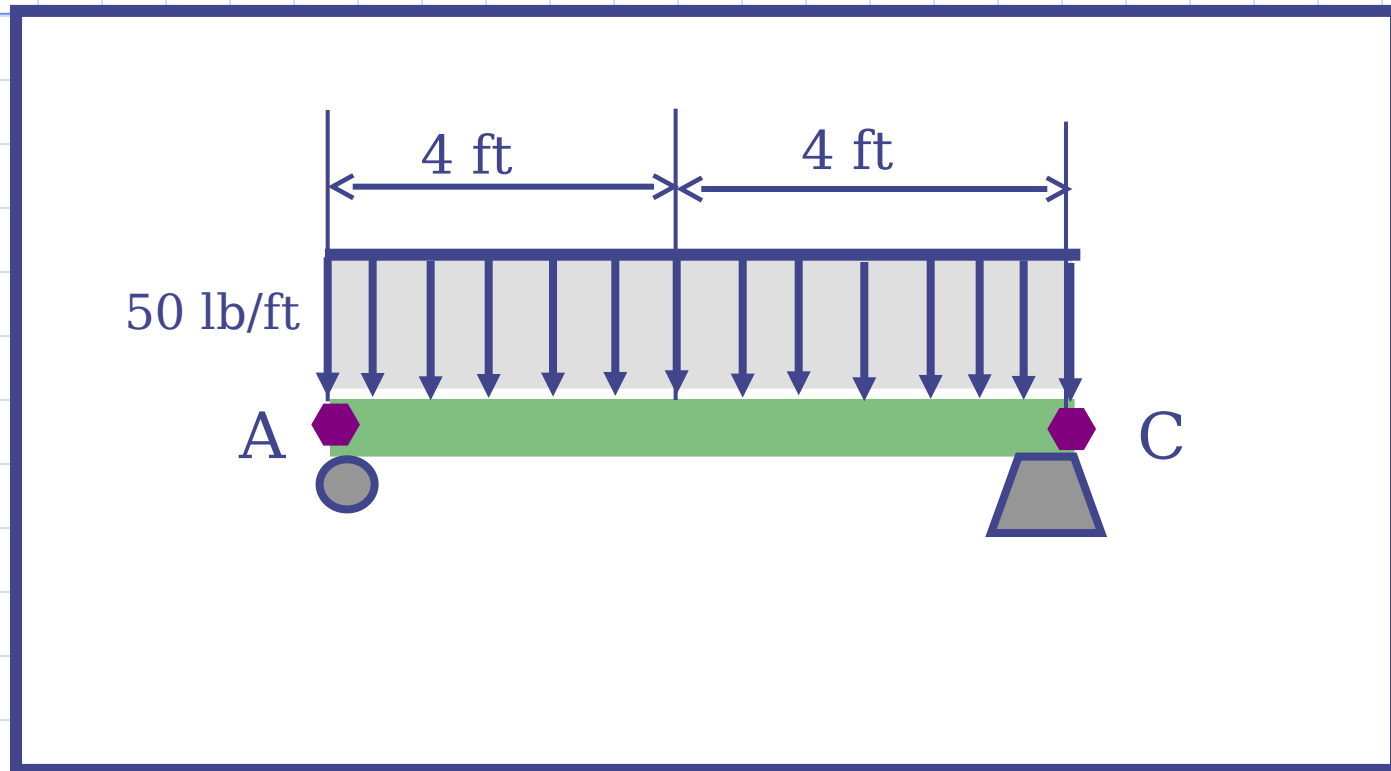
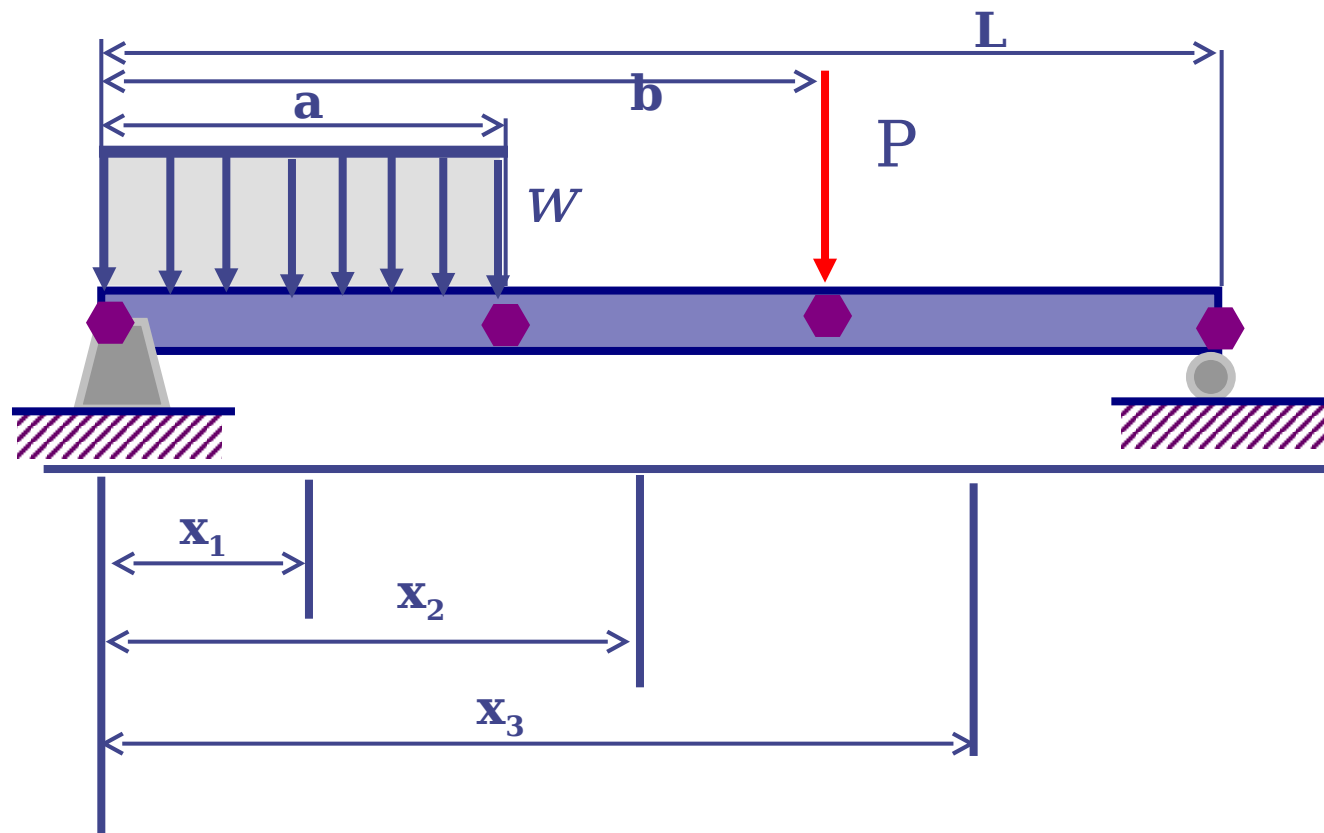
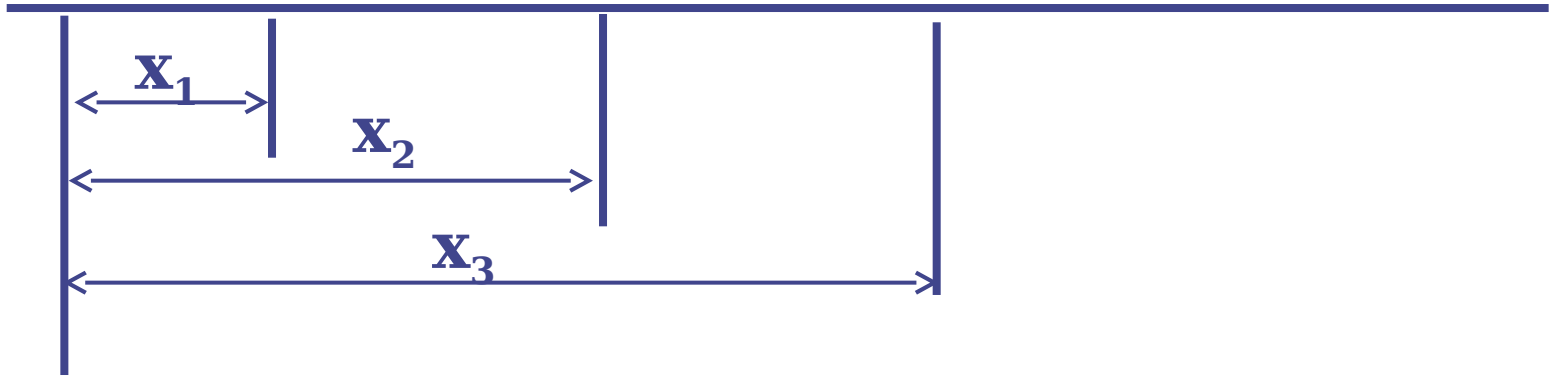
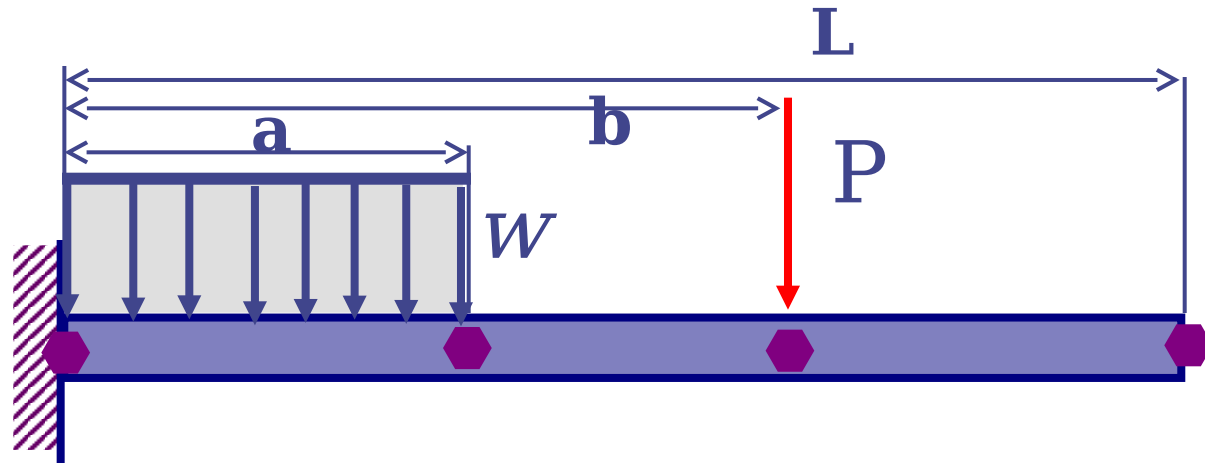
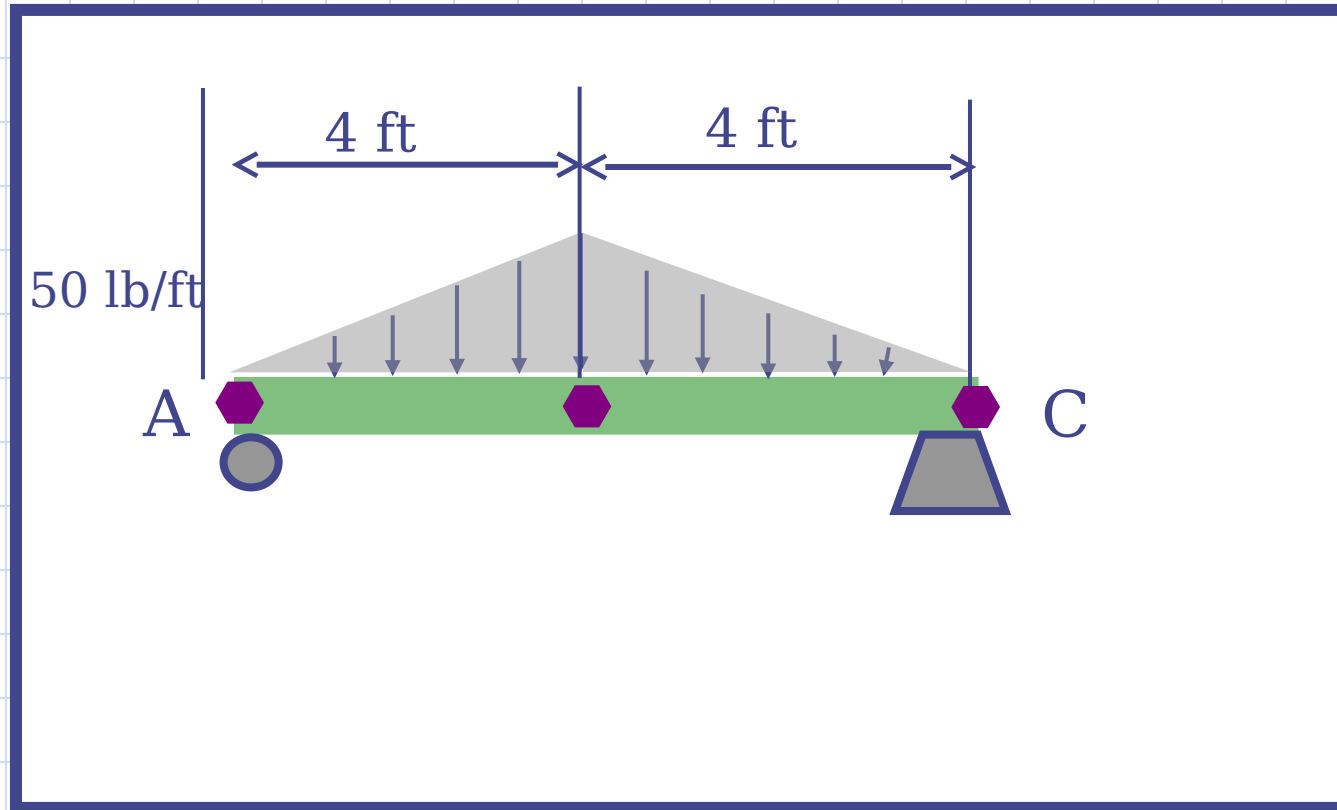
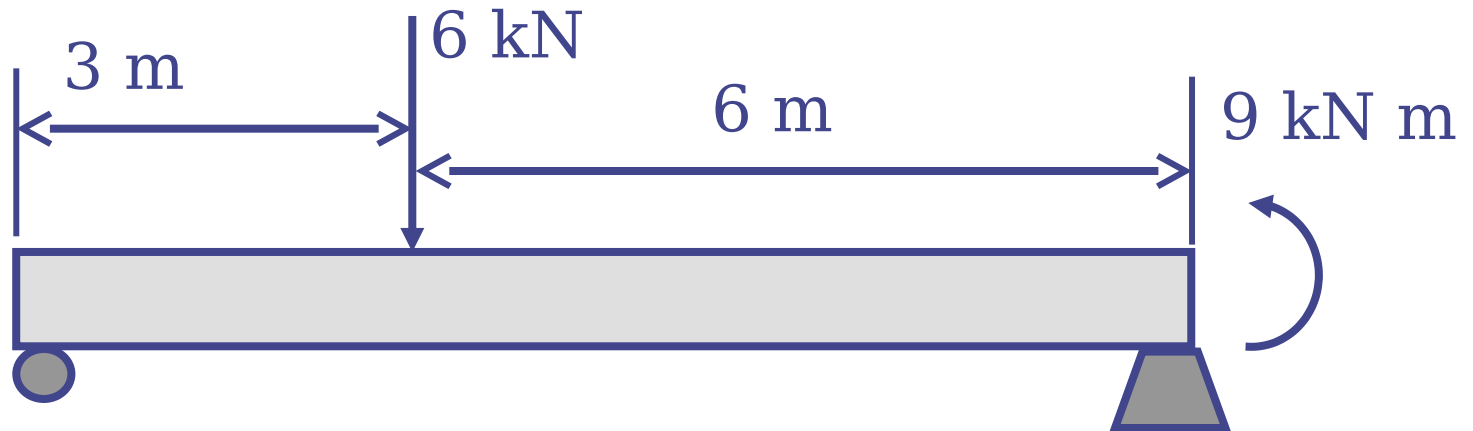
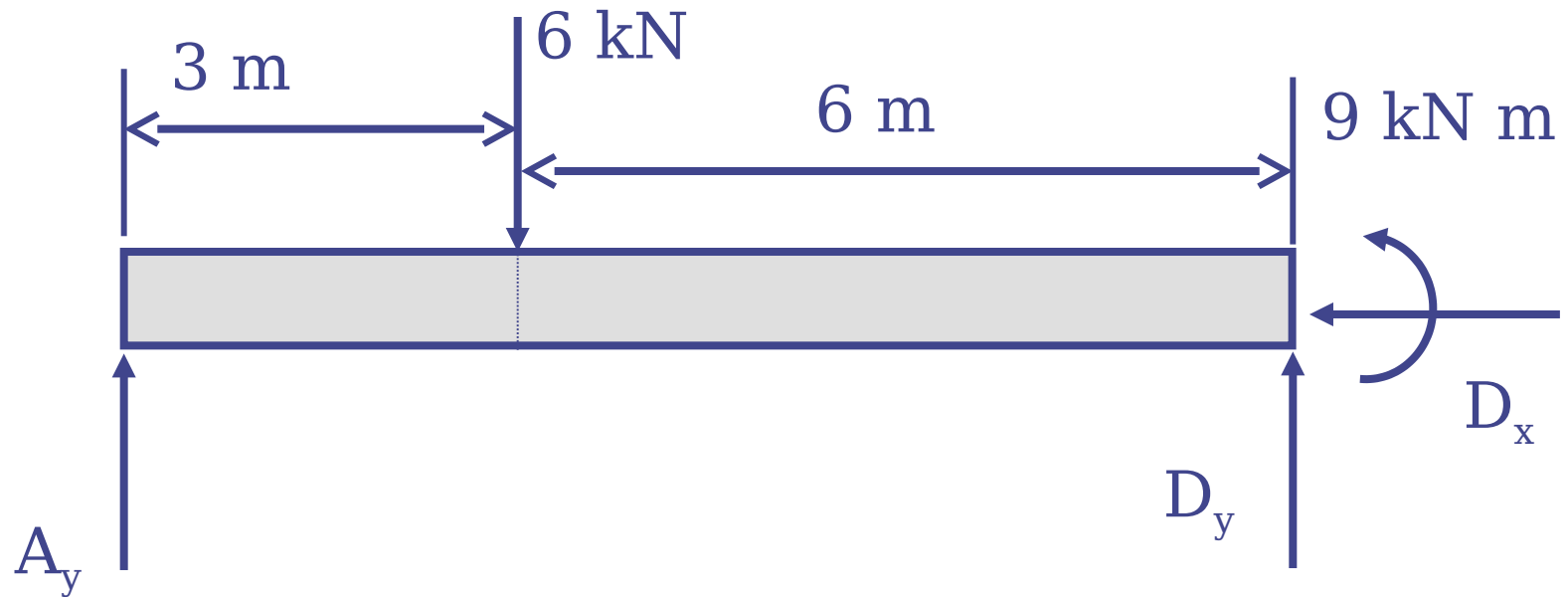
Positive Moment

Positive Moment

V
V
V
V
Positive shear
M
M
Positive moment
M
M
Beam sign convention
Figure 07.11

Critical Points

Determine the internal forces just to the left and the right of the external force

Critical Points

L
b
a
P
W
x1
x2
x3

L
b
a
P
W
x1
x2
x3

Critical Points

Shear Force and Bending Moment Diagram

FBD of Beam

$$\sum F_x = 0$$

$$D_x = 0$$

$$\sum F_y = 0$$

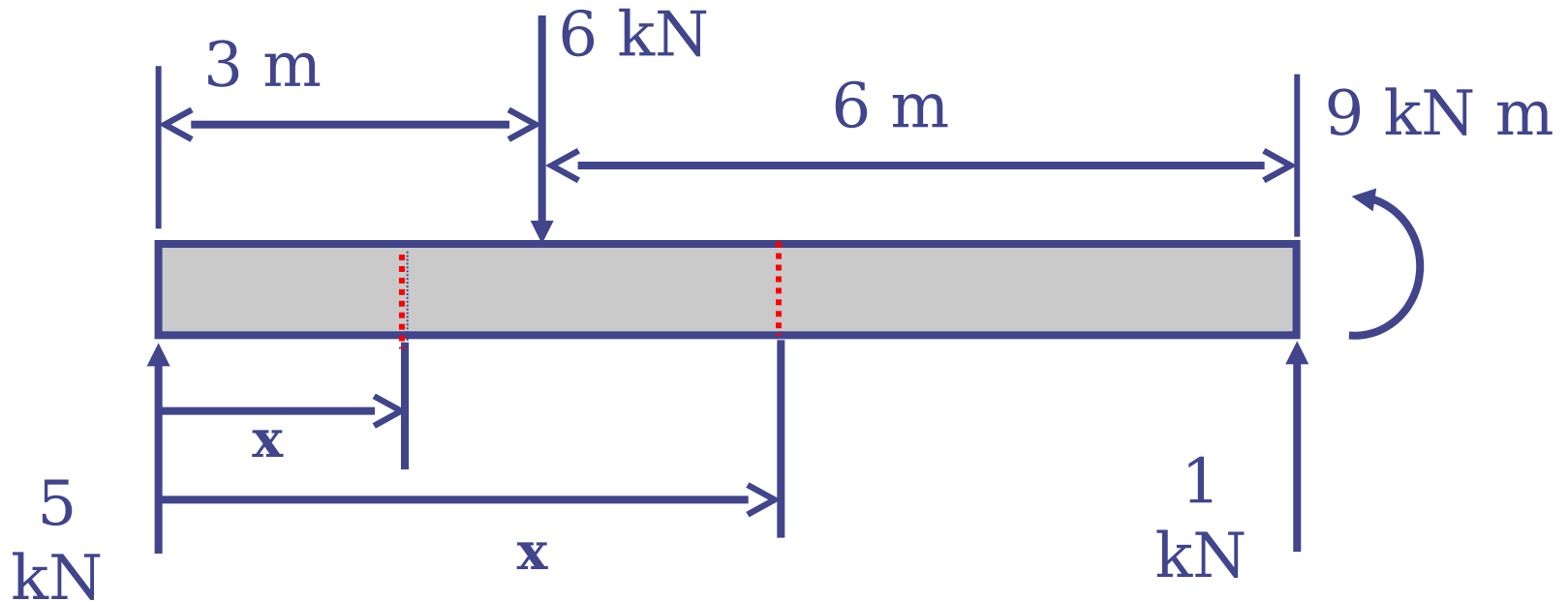
$$A_y - 6 + D_y = 0$$

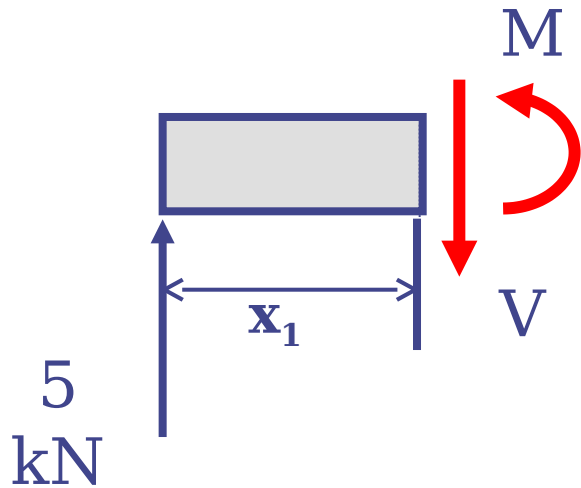
$$\sum M_D = 0$$

$$9 + 6(6) - A_y(9) = 0$$

$$A_y = 5\,kN$$

$$D_y = 1\,kN$$

FBD of Beam

$$0 \leq x < 3$$

$$3 < x \leq 9$$

$$0 \le x < 3$$

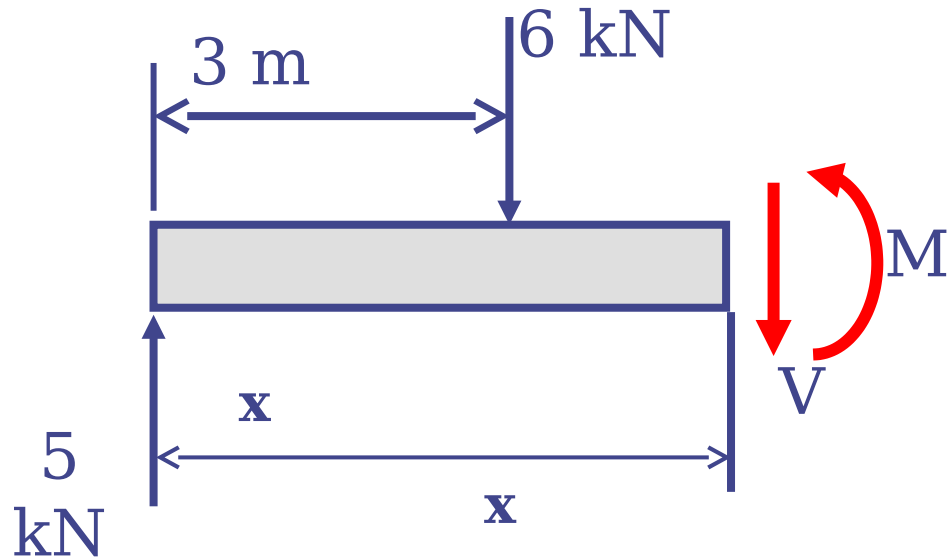
$$\sum F_y = 0$$

$$5 - V = 0$$

$$V = 5\ \text{kN}$$

$$\sum M = 0$$

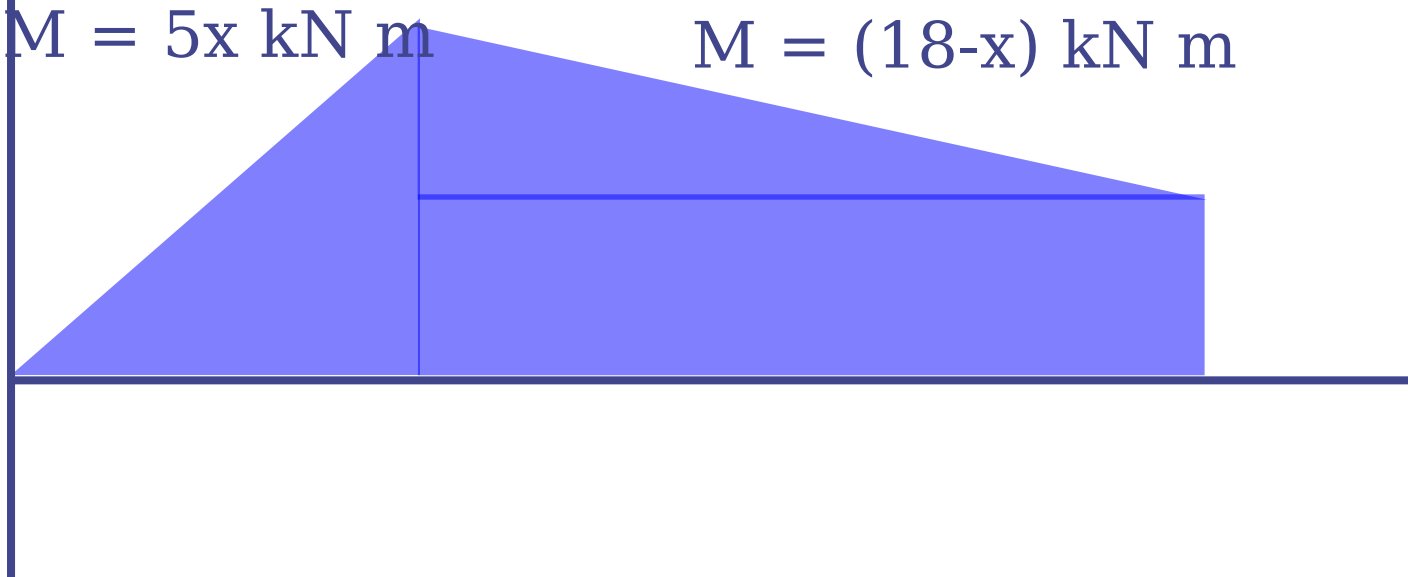
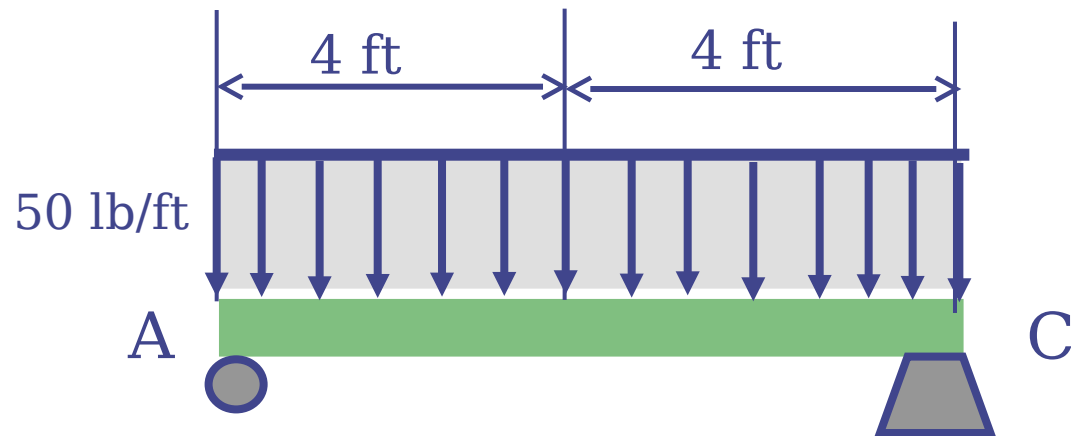
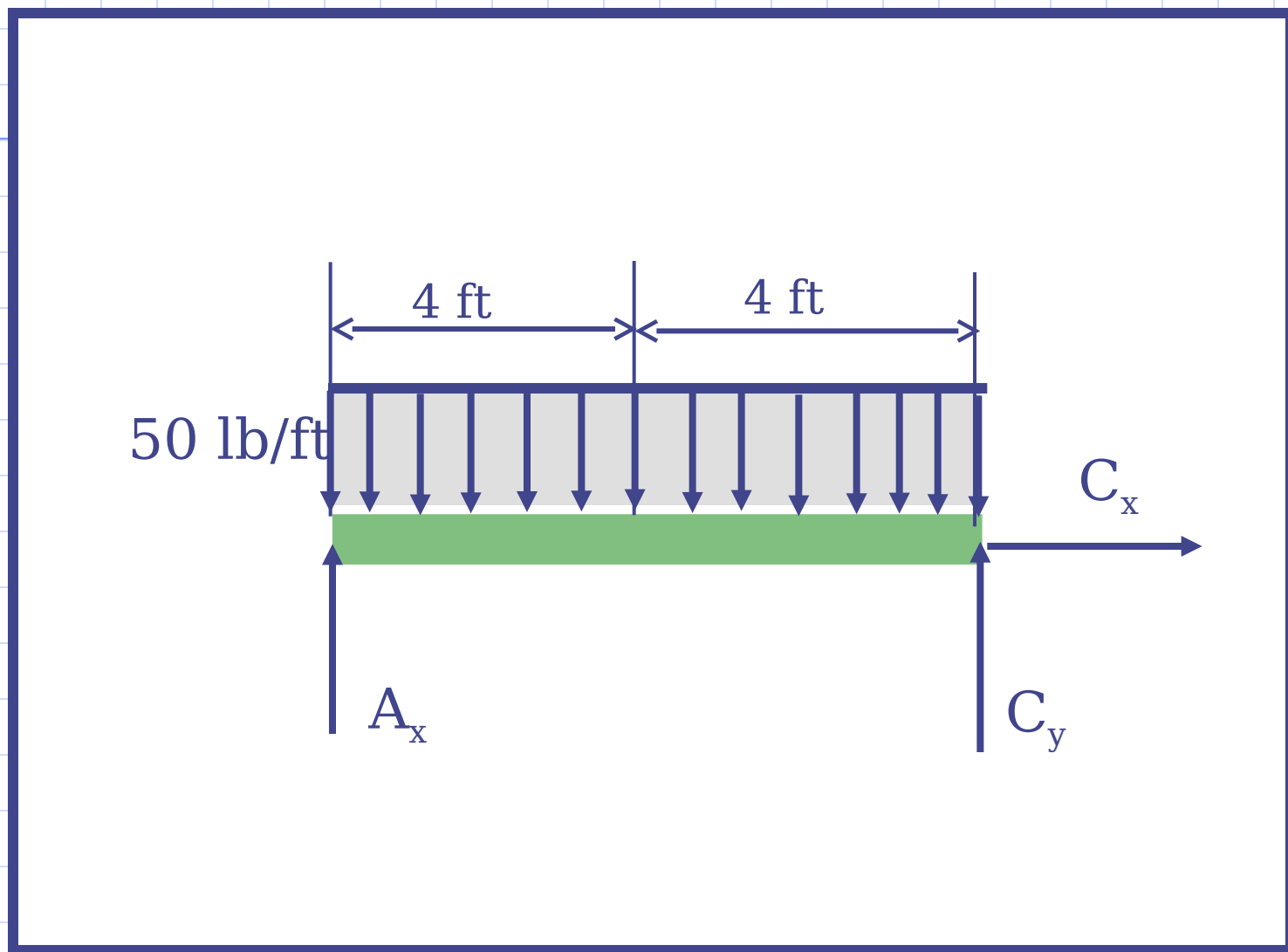
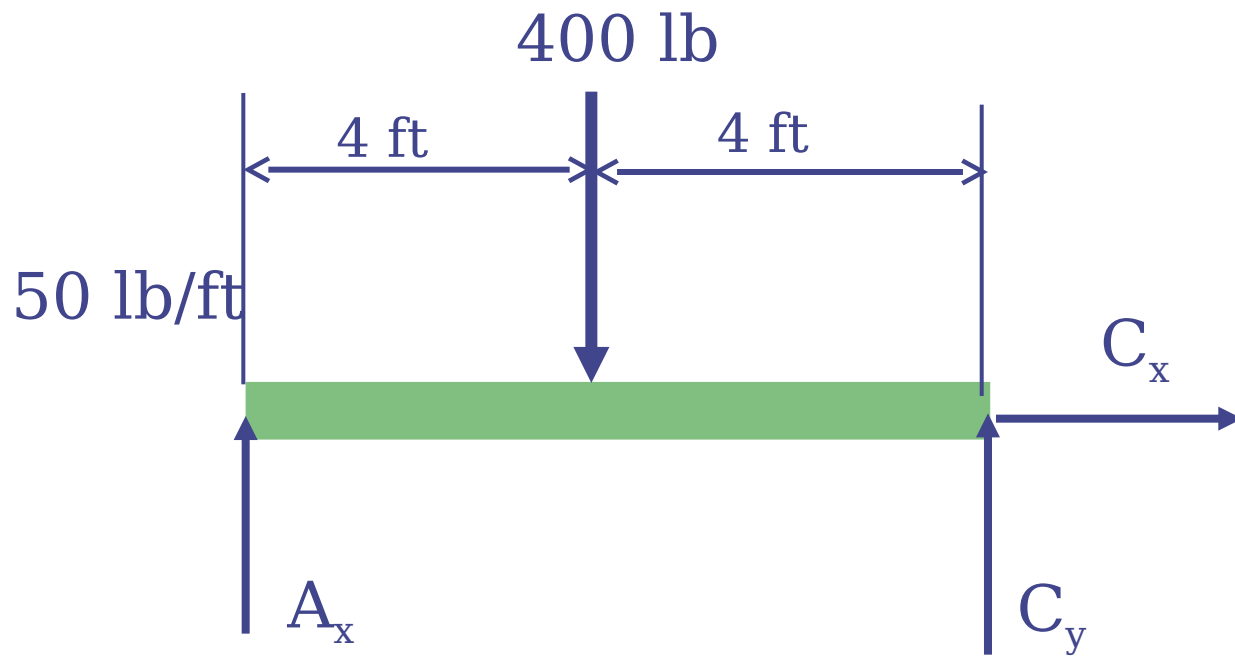
$$M - (5\ \text{kN})\,x = 0$$

$$M = 5x\ \text{kN} \cdot \text{m}$$

6 kN
3 m
M
V
x
5
kN
x
3< x ≤9
∑Fy =0
5 kN- 6 kN- V =0
V =- 1 kN
∑M =0
M- (5)x + (6)(x - 3) =0
M =(18- x) kN ·m

V (kN)
V = 5 kN
V = -1 kN
M = 5x kN m
M = (18-x) kN m
M (kN m)

4 ft
4 ft
50 lb/ft
A
C

4 ft
4 ft
50 lb/ft
C
x
A
x
C
y

400 lb
4 ft
4 ft
50 lb/ft
C_x
A_x
C_y

$$\sum F_x = 0$$

$$C_x = 0$$

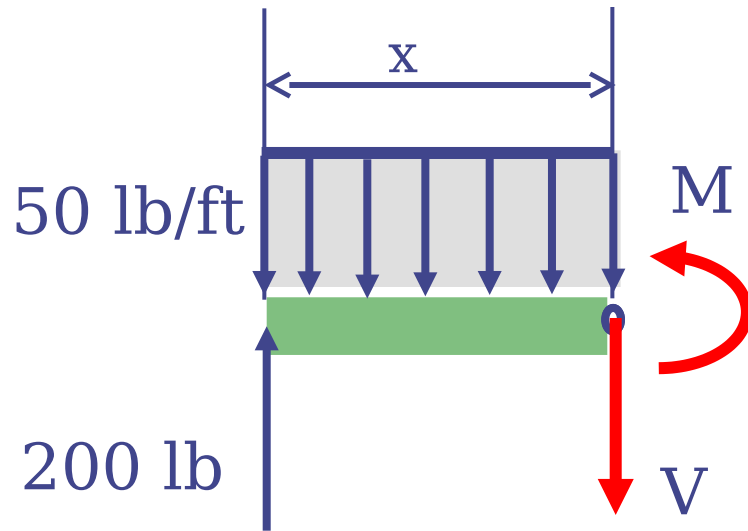
$$\sum F_y = 0$$

$$A_y + C_y - 50(8) = 0$$

$$\sum M_A = 0$$

$$50(8)(4) - C_y(8) = 0$$

$$A_y = 200 \text{ lb}$$

$$C_y = 200 \text{ lb}$$

$$0 \le x \le 8$$

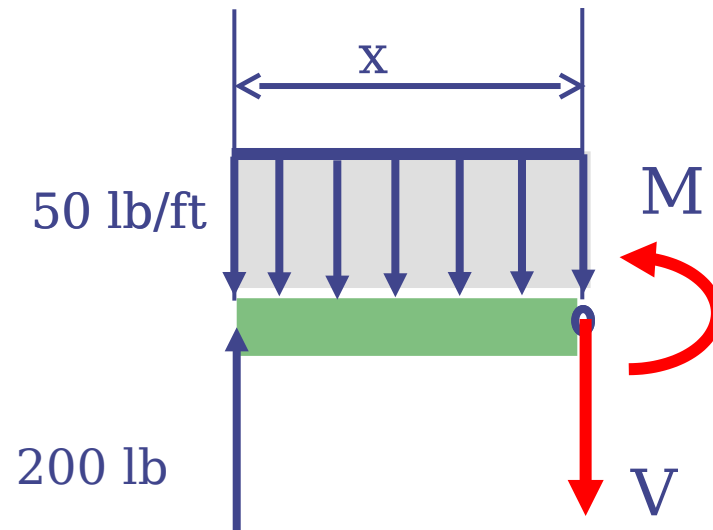
$$\sum F_y = 0$$

$$-V + 200\ \text{lb} - 50x = 0$$

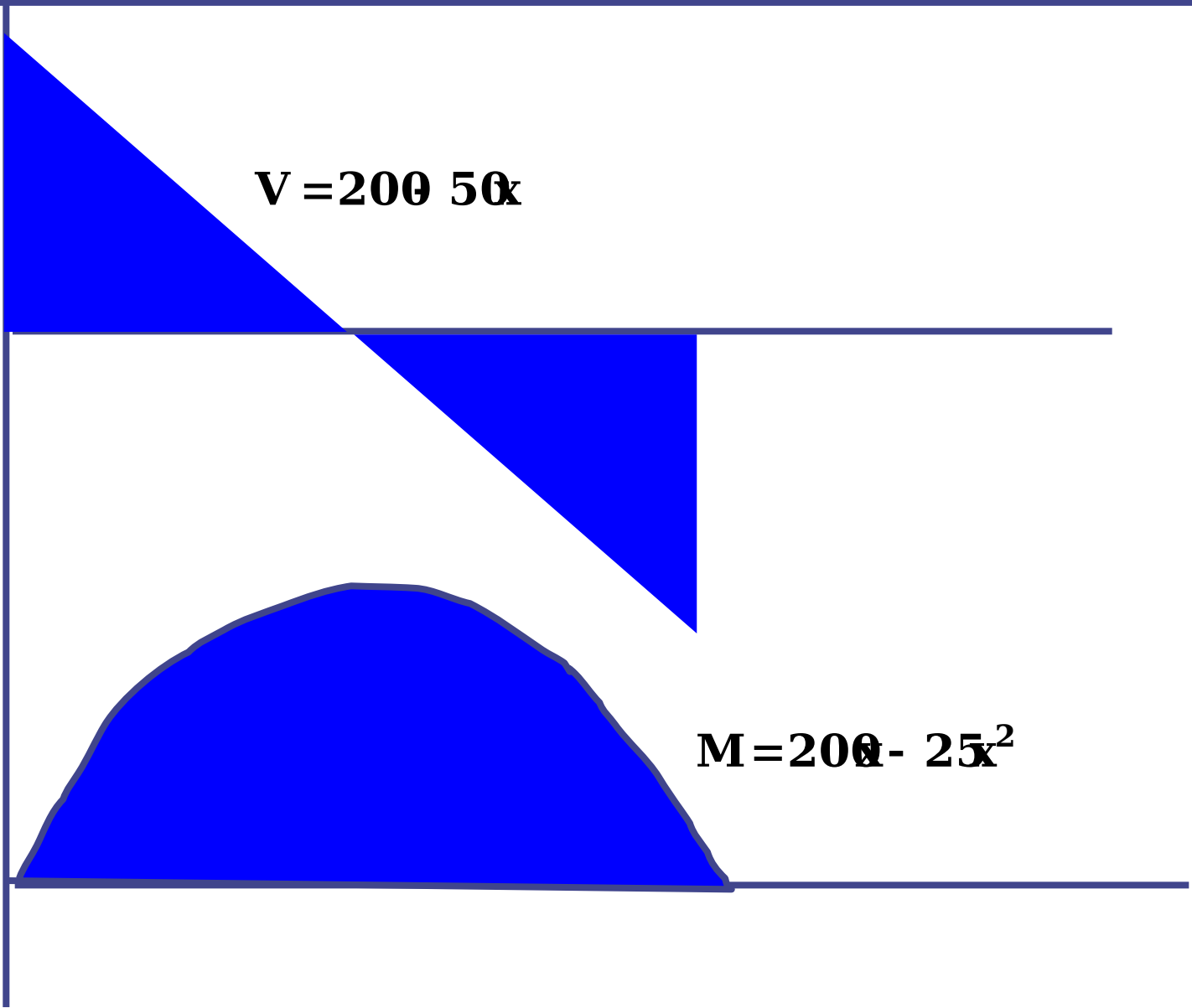
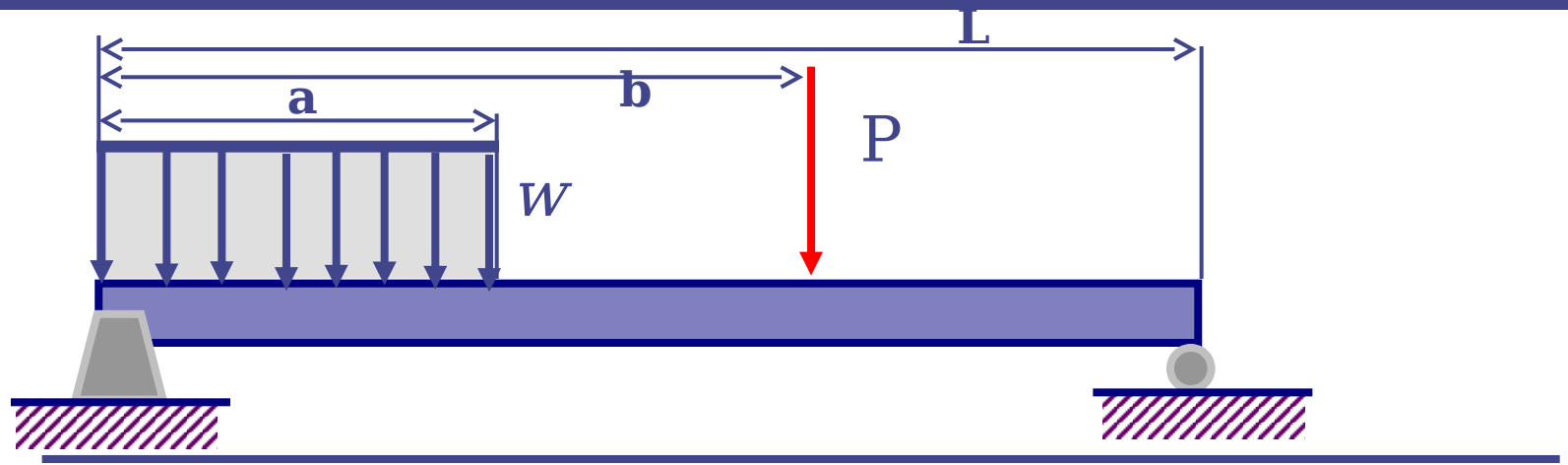
$$V = \left(50x - 200\right)\text{lb}$$

$$\sum M = 0$$

$$M - 200x + 50(x)\left(\frac{x}{2}\right) = 0$$

$$M = 200x - 25x^2\ \ \text{lb}\cdot\text{ft}$$

$$at\, x = 0, \quad V = 200\, lb \quad M = 0$$

$$at\, x = 8, \quad V = -200\, lb \quad M = 0$$

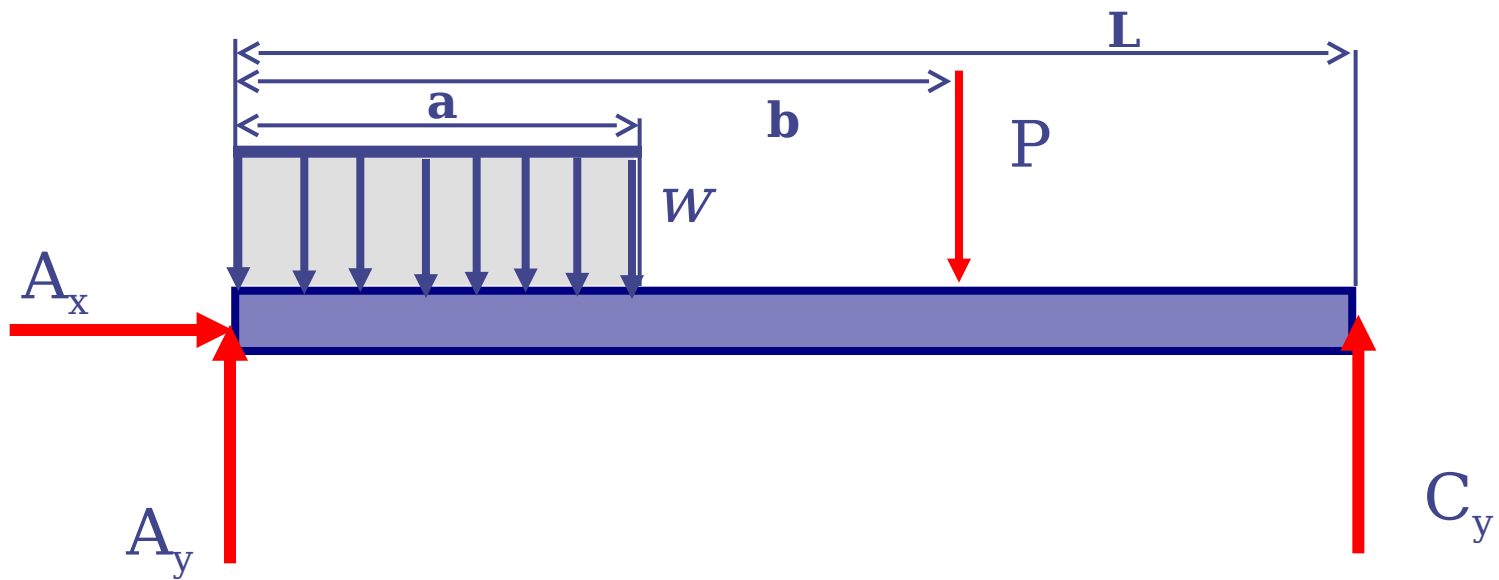
V (lb)
V = 200 - 50x
M (lb ft)
M = 200x - 25x2

$w = 100$ lb/ft

$L = 15$ ft

$a = 5$ ft

$b = 10$ ft

$P = 1000$ lb

L
a
b
P
w
A_x
A_y
C_y

$$\sum F_x = 0$$

$$A_x = 0$$

$$\sum F_y = 0$$

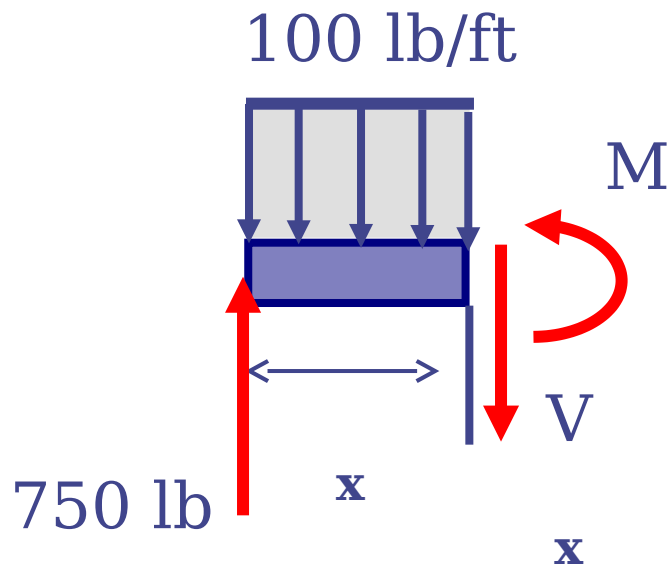
$$A_y + C_y - 100(5) - 1000 = 0$$

$$A_y + C_y = 1500$$

$$\sum M_A = 0$$

$$-100(5)(2.5) - 1000(10) + C_y(15) = 0$$

$$A_y = 750 \text{ lb}$$

$$C_y = 750 \text{ lb}$$

$$0 \le x \le 5$$

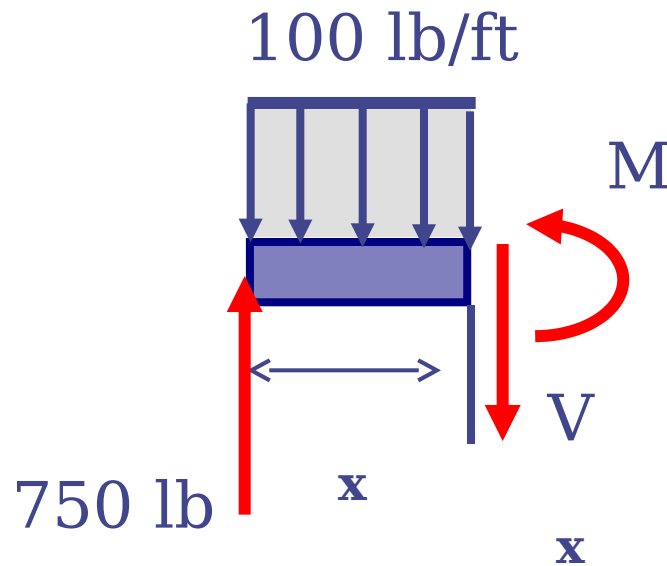
$$\sum F_y = 0$$

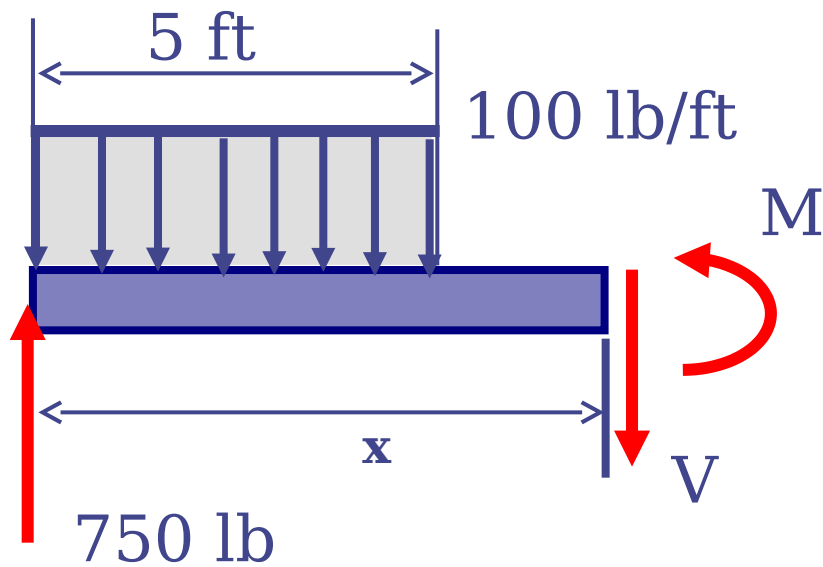
$$-V + 750\text{ lb} - 100x = 0$$

$$V = (750 - 100x)\text{ lb}$$

$$\sum M = 0$$

$$M - 750x + 100(x)\left(\frac{x}{2}\right) = 0$$

$$M = \left(750x - 50x^2\right)\text{ lb}\cdot\text{ft}$$

$$at\ x = 0, \quad V = 750\ lb \quad M = 0$$

$$at\ x = 5, \quad V = 250\ lb \quad M = 2500\ lb \cdot ft$$

$$5 \le x < 10$$

$$\sum F_y = 0$$

$$-V + 750\ \text{lb} - 100(5) = 0$$

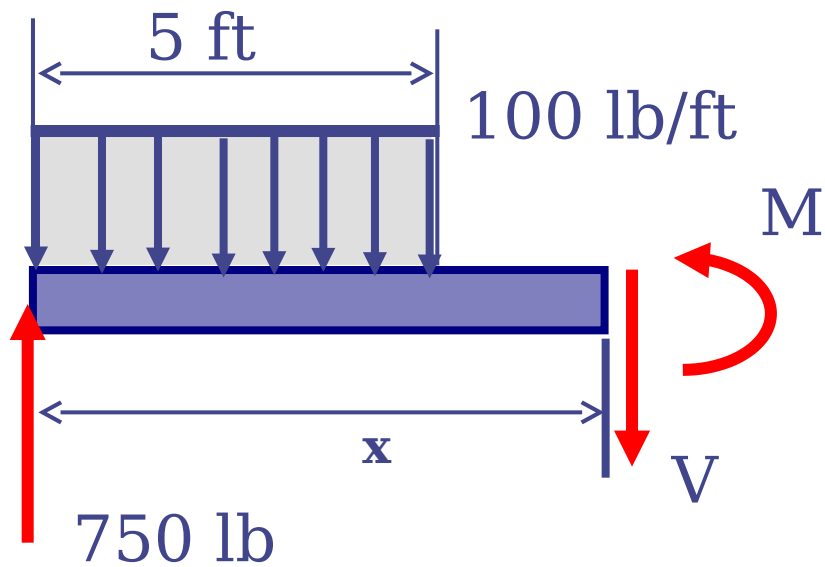
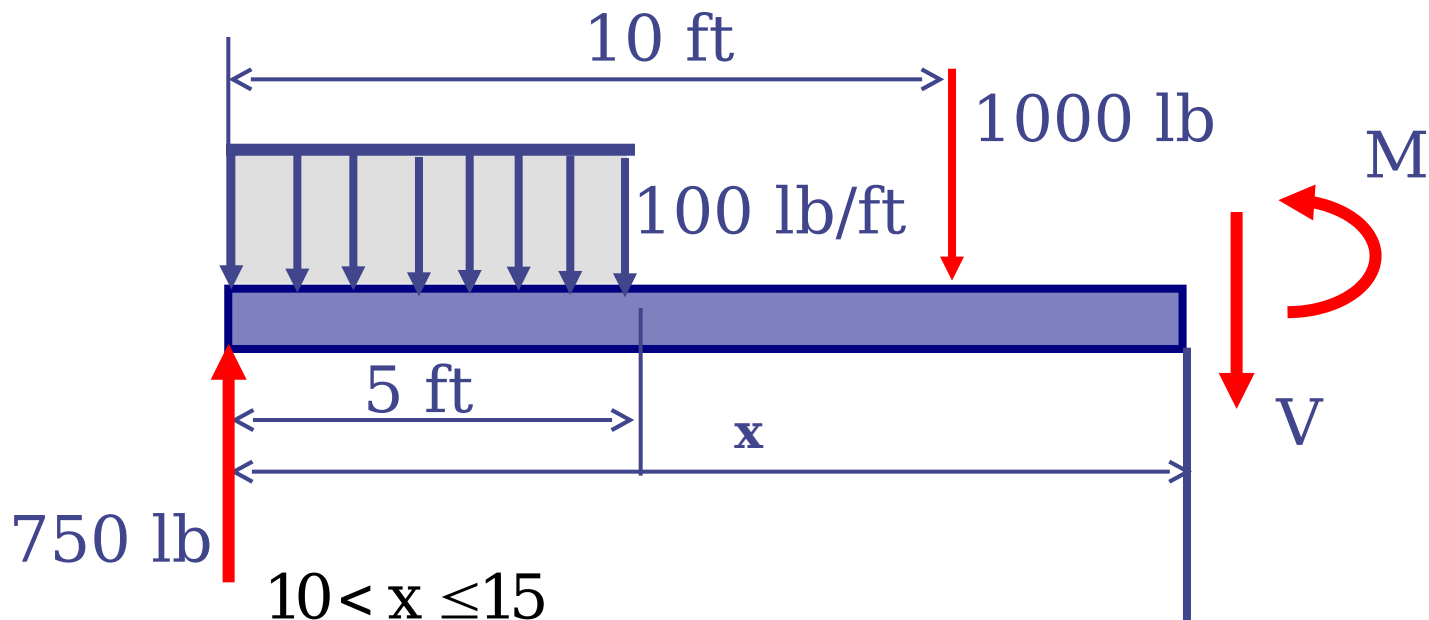
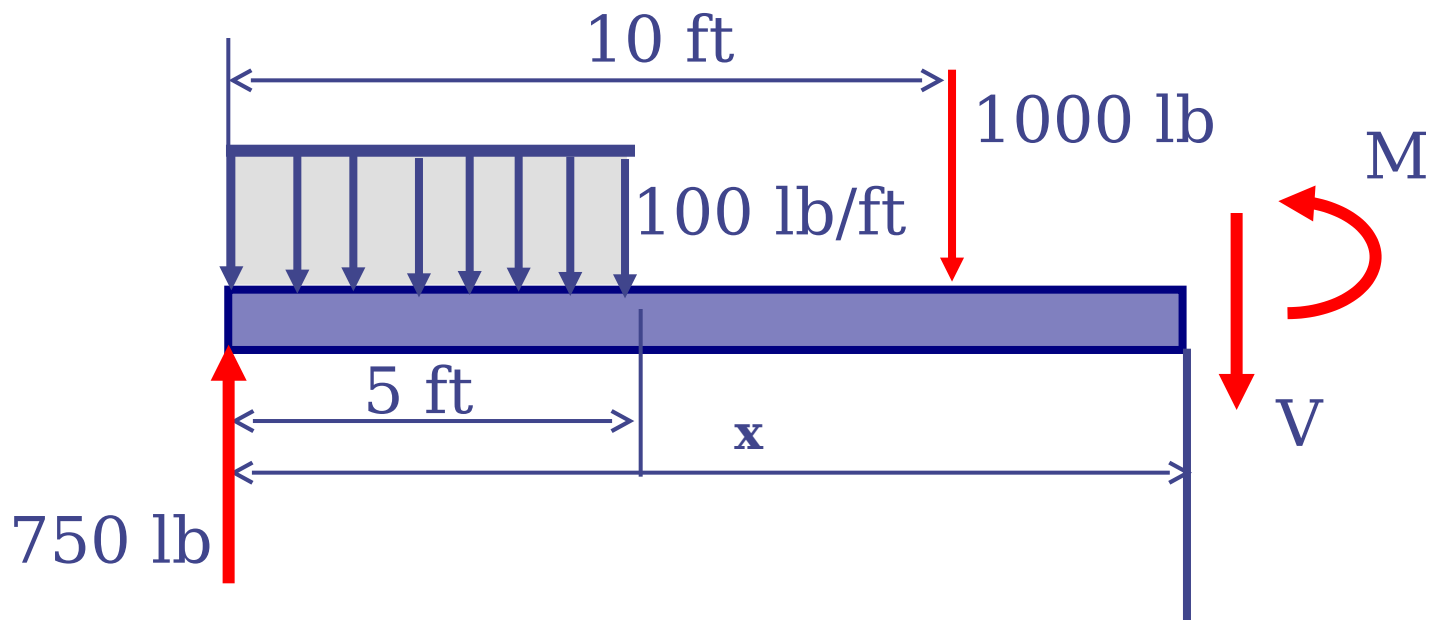
$$V = 250\ \text{lb}$$

$$\sum M = 0$$

$$M - 750x + 100(5)\left(x - 2.5\right) = 0$$

$$M = \left(750x - 500x + 1250\right)\ \text{lb}\cdot\text{ft}$$

$$M = \left(250x + 1250\right)\text{lb}\cdot\text{ft}$$

$$at\ x = 5, \quad V = 250\ lb \quad M = 2500\ lb \cdot ft$$

$$at\ x = 10, \quad V = 250\ lb \quad M = 3750\ lb \cdot ft$$

10 ft
1000 lb
M
100 lb/ft
5 ft
x
V
750 lb
$10 < x \leq 15$
$\sum F_y = 0$
$-V + 750\ \text{lb} - 100(5) - 1000 = 0$
$V = -750\ \text{lb}$
$\sum M = 0$
$M - 750x + 100(5)\left(x - 2.5\right) + 1000\ (x - 10) = 0$
$M = \left(-750x\ + 11250\right)\ \text{lb} \cdot \text{ft}$

$$\text{at } x = 10^+, \quad V = -750\ lb \quad M = 3750\ lb \cdot ft$$

$$\text{at } x = 15, \quad V = -750\ lb \quad M = 0\ lb \cdot ft$$

$$0 \le x \le 5$$

$$V = 750 \text{ lb} - 100 x$$

$$M = 750 x - 50 x^2 \text{ lb} \cdot \text{ft}$$

$$5 \le x < 10$$

$$V = 250 \text{ lb}$$

$$M = (250 x + 1250) \text{ lb} \cdot \text{ft}$$

$$10 < x \le 15$$

$$V = -750 \text{ lb}$$

$$M = (-750 x + 11250) \text{ lb} \cdot \text{ft}$$

V (lb)
V = 750 lb - 100x
V = 250 lb
V = - 750 lb
M = (250x + 1250) lb·ft
M = 750x - 50x2 lb·ft
M (lb ft)
M = (- 750x + 11250) lb·ft

V
1000
500
0
-500
-1000
0.0
5.0
10.0
15.0
V
M
4000
3500
3000
2500
2000
1500
1000
500
0
0.0
5.0
10.0
15.0
M

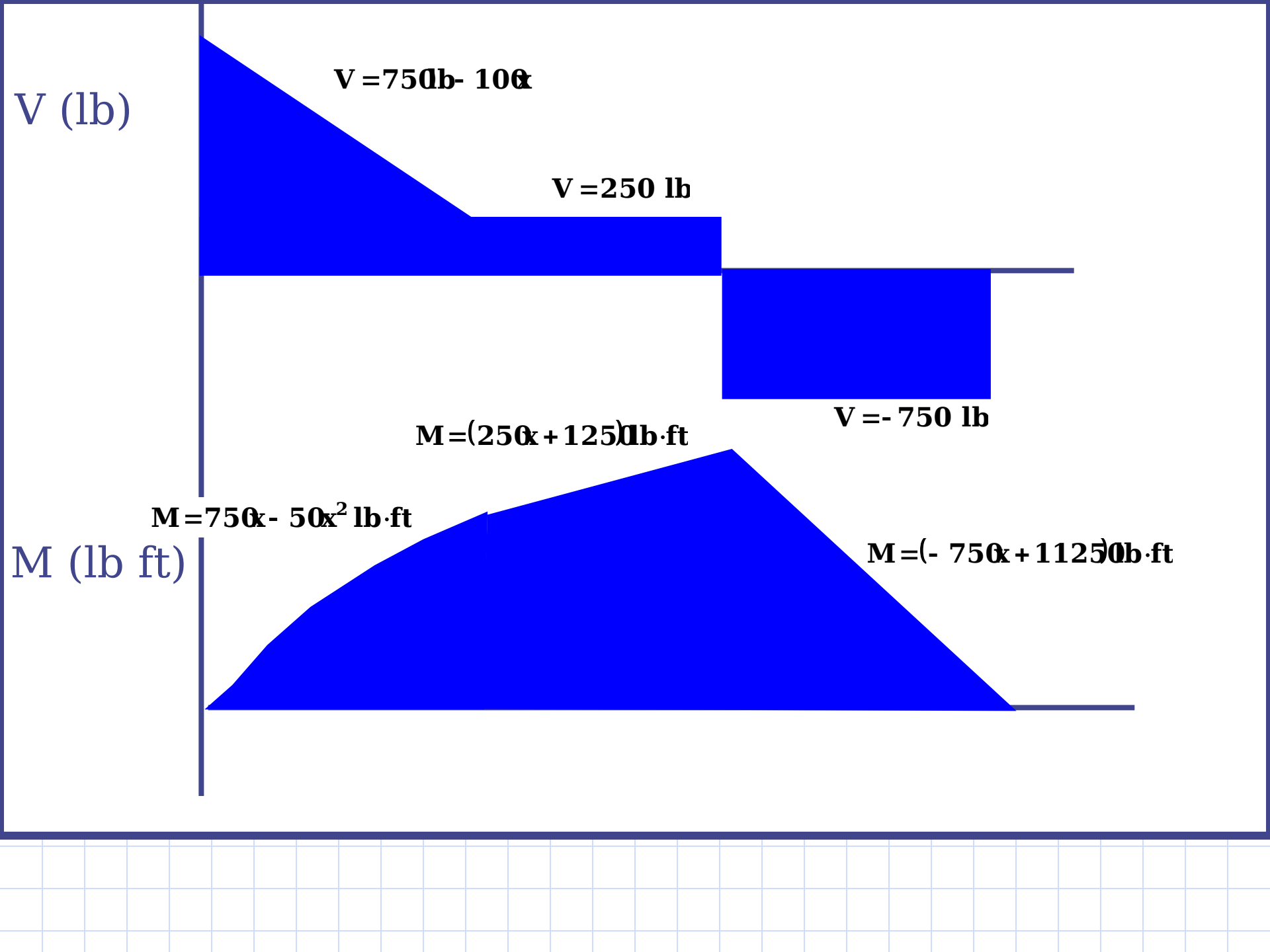
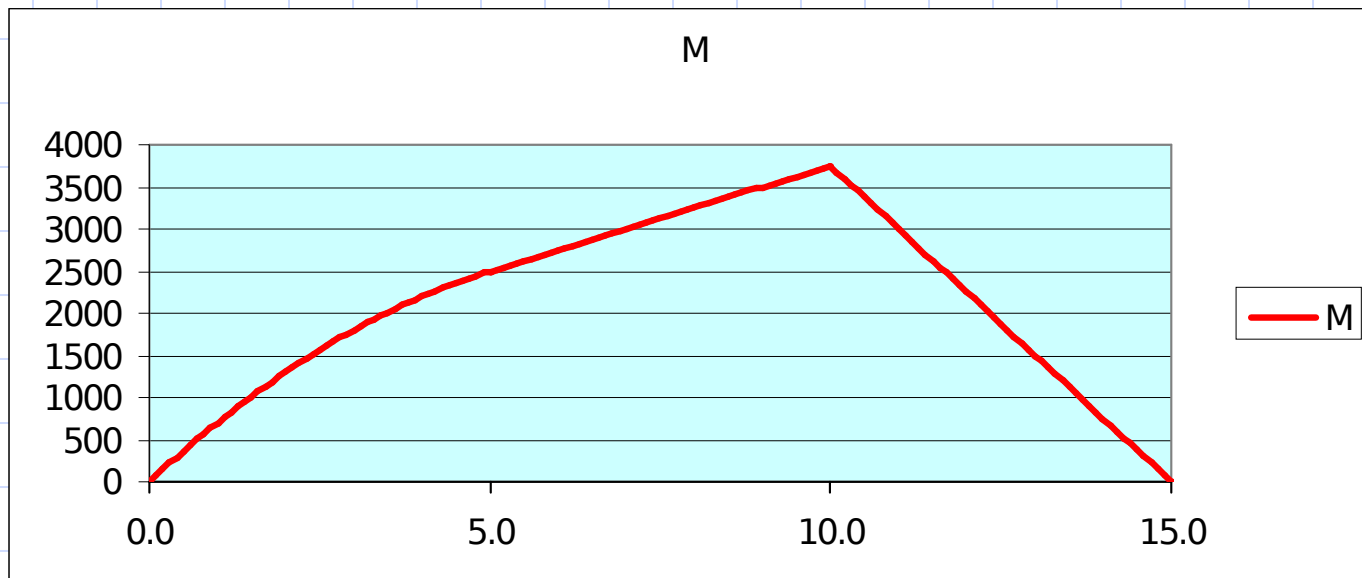
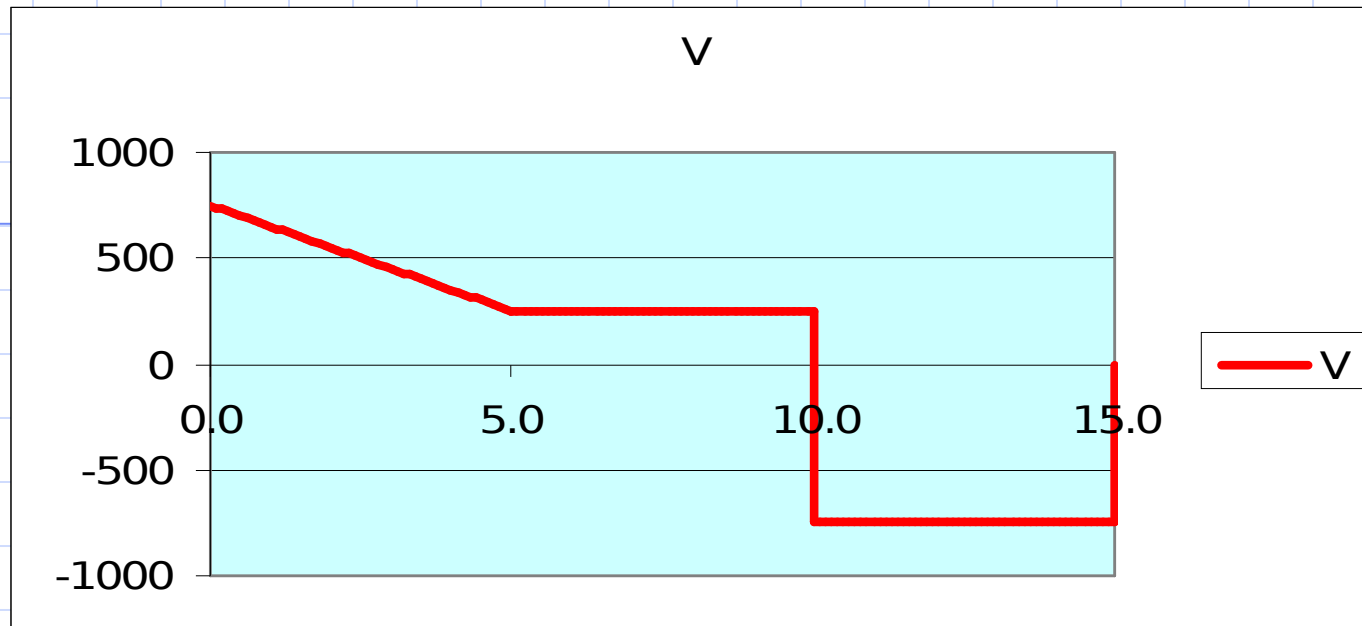
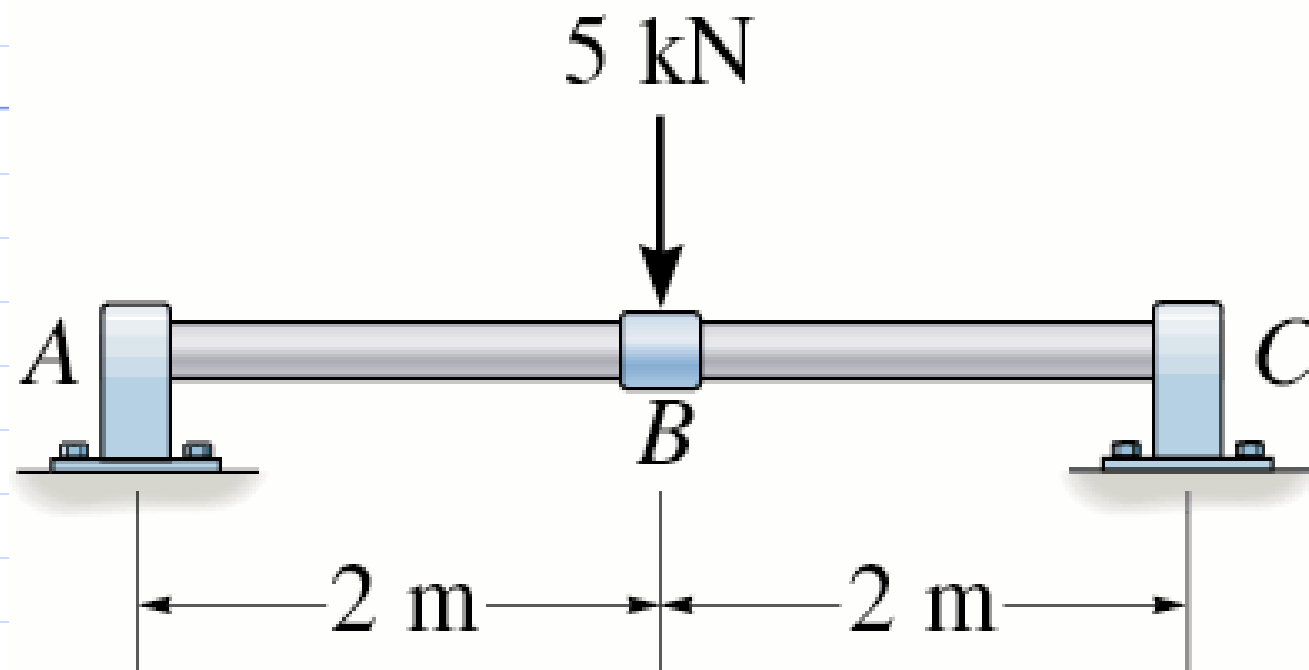
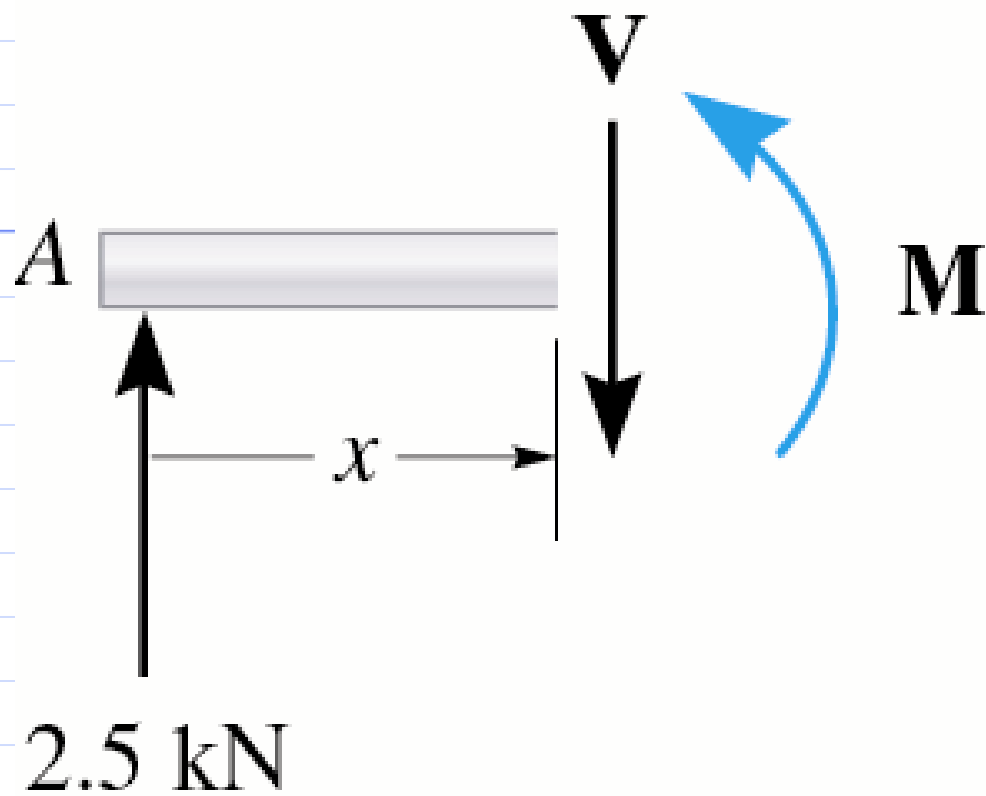



Figure 07.12(a)

Figure 07.12(b)

Figure 07.12(c)

5 kN
A
B
C
2.5 kN
2.5 kN
V (kN)
V = 2.5
x
V = –2.5
M (kN·m)
M = 2.5x
Mmax = 5
M = (10 – 2.5x)
x

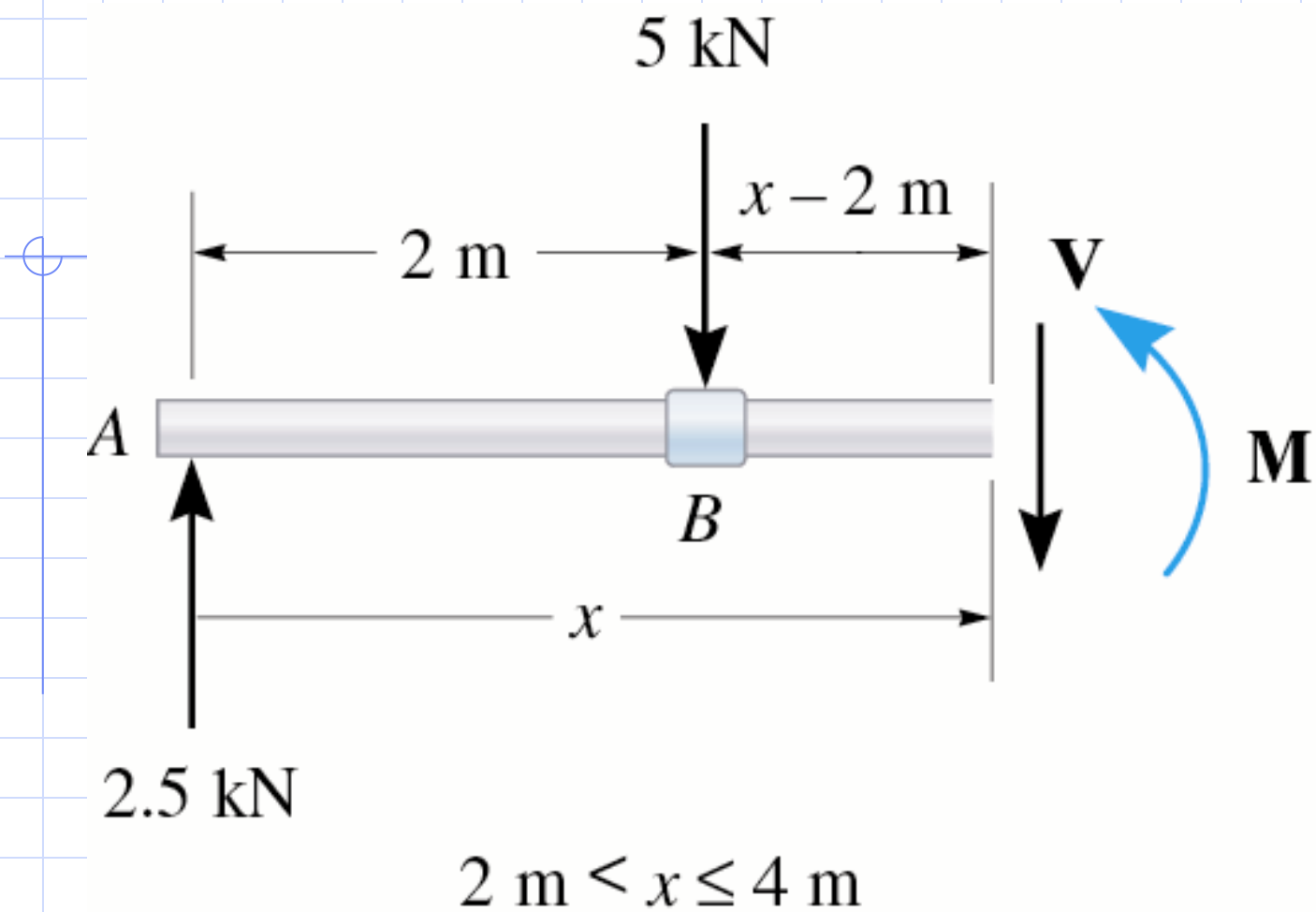
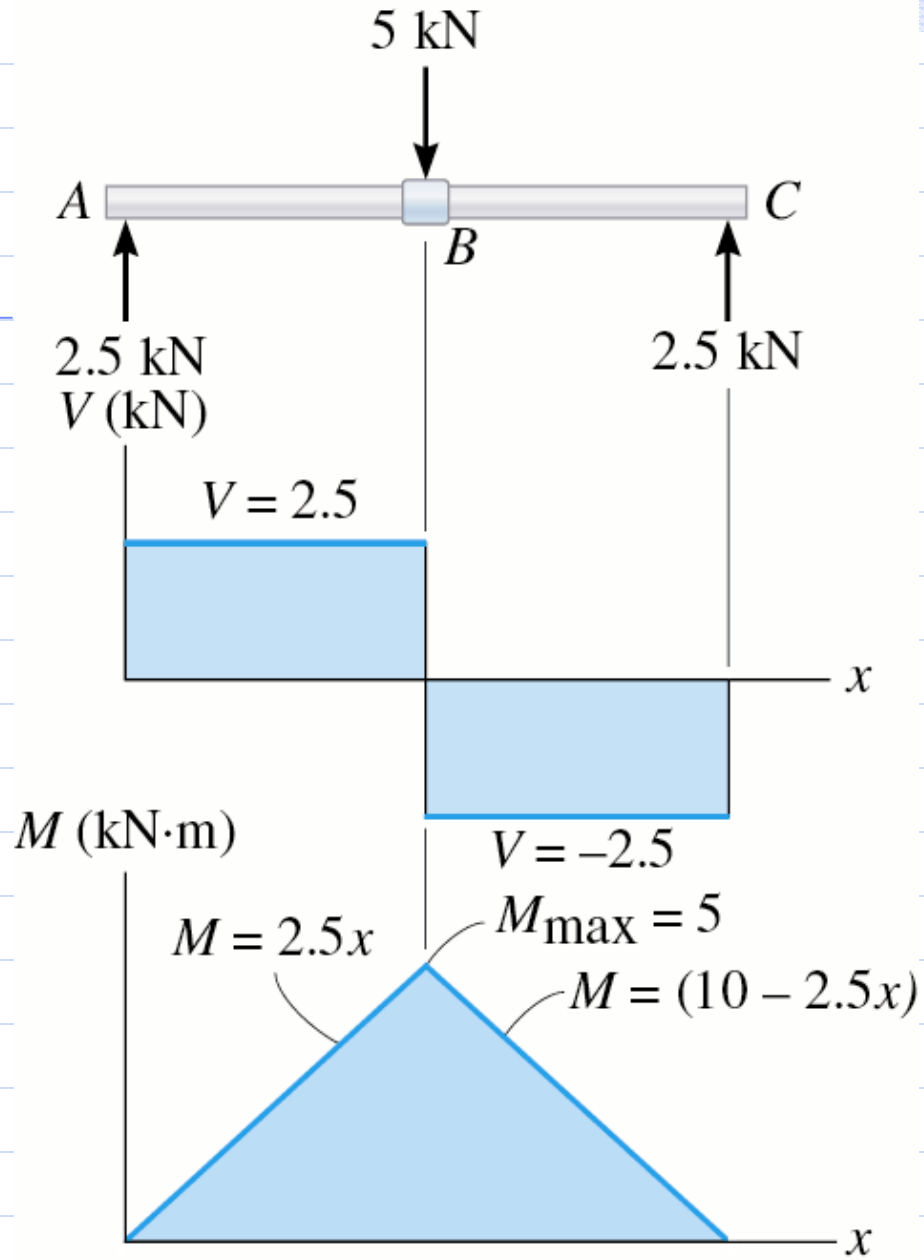
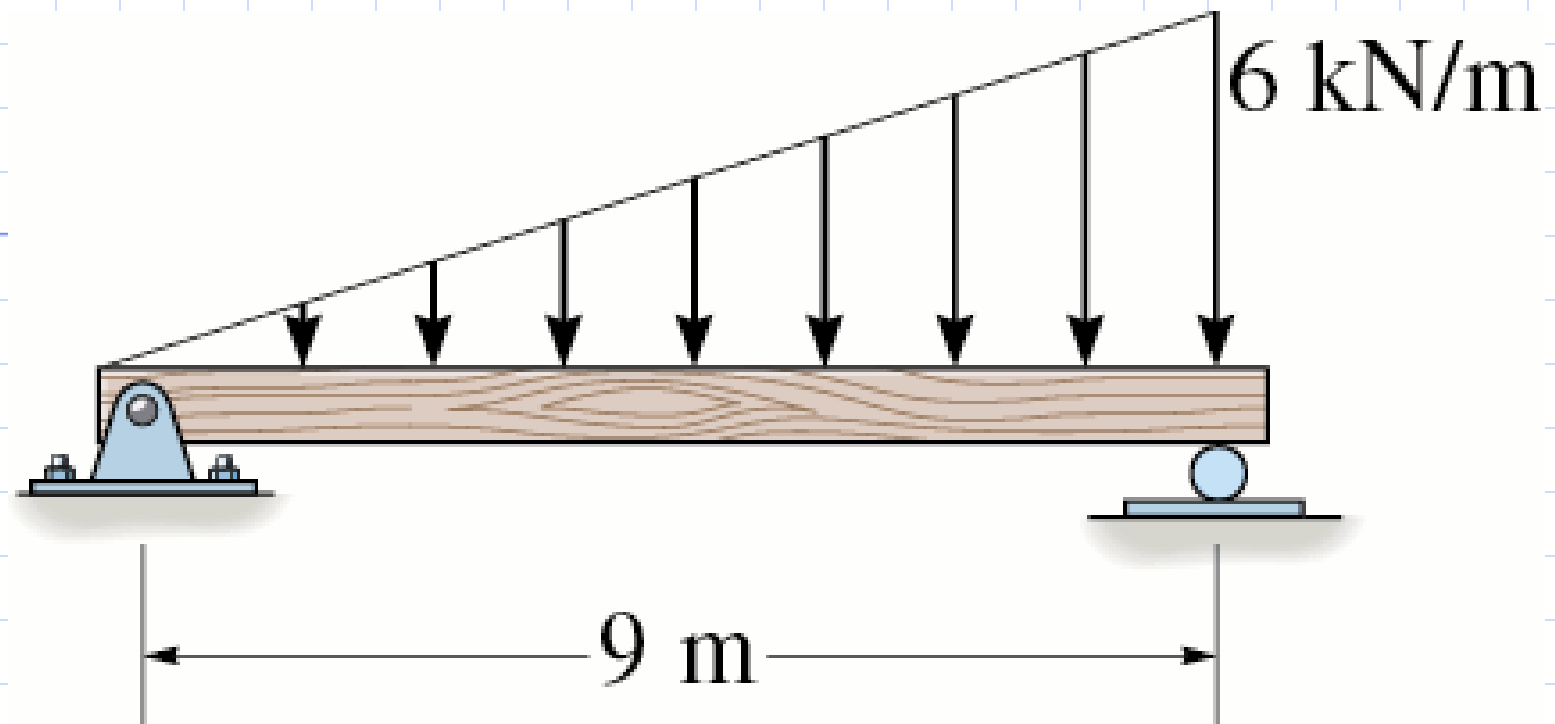



Figure 07.13(a)

Figure 07.13(b)

6 kN/m
9 kN
18 kN
V (kN)
9
V = 9 − x²/3
x
5.20 m
−18
M (kN·m)
M = 9x − x³/9
Mmax = 31.2
x

Prob 07.49

2 kN/m
50 kN · m
A
B
C
5 m
5 m
Prob 07.51

Prob 07.59